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ABSTRACT. A strong point explosion is examined in an exponential atmosphere without consideration of the real properties of the air. A numerical solution of the point equation of gas dynamics is given and the computation is extended to later phases.

An approximate law for the propagation of a shock wave front in the case of an atmosphere whose density depends exponentially on altitude was obtained in [1] for the simplest model of a powerful explosion. This law was further specified in [2], where the dependence of pressure, density and the speed of the particles in the front on time and the angular coordinate were also determined. An approximate calculation of the two-dimensionality of the phenomenon was conducted in [3]. The authors proceeded on the basis of an assumption of local radiality of flow; as a result, the problem was converted to one-dimensionality with parametric dependence of the solution on the angular coordinate. /25*

A relatively late stage of a plane explosion was examined in [4, 5]. The asymptotic self-modeling solutions that were obtained were applied to a point explosion [6]. Similar asymptotic considerations were made in [7, 8]. The first attempt to investigate the problem numerically in a point formulation was made in 1955 in [9]; see also [14].

In the present paper, the problem is considered in the same initial formulation as in the preceding papers; a strong point explosion is examined in an exponential atmosphere without consideration of the real properties of the air. Unlike previous reports, however, where the motion was considered either by approximate methods or for early moments in time, when the non-homogeneity was not very powerfully manifested, the present paper contains a numerical solution of the point equations of gas dynamics and the computation is extended to later phases. The results of the calculation are

*Numbers in the margin indicate pagination in the foreign text.

compared with data in [2].

1. Statement of the Problem. Let us consider a non-viscous, complete gas, whose thermal conductivity and radiation are not taken into account. The density ρ_0' and pressure p_0' are exponentially dependent on the altitude z', reckoned from the point P_0, where at the initial moment $t = 0$ the energy E_0 is liberated.

$$\rho_0' = \rho_{00}' \exp\frac{-z'}{\Delta}, \qquad p_0' = p_{00}' \exp\frac{-z'}{\Delta} \tag{1.1}$$

Here Δ is the scale of non-homogeneity [6]. In the explosion, a shock wave is produced which separates the region of flow of the excited gas from the unexcited portion. The phenomenon possesses axial symmetry, and all of its characteristics depend on the cylindrical coordinates z', r', and the time t'. The motion is viewed in the half plane $\Pi(r' \geqslant 0)$, bounded by the axis of symmetry. Let p' represent the pressure, ρ' the density, and u' and v' the horizontal and vertical components of the speed, respectively. The dimensionless variables are introduced according to the formulas

$$\begin{gathered} t = \frac{t'}{(\rho_{00}'\Delta^5 / E)^{1/2}}, \quad z = \frac{z'}{\Delta}, \quad r = \frac{r'}{\Delta}, \quad \rho = \frac{\rho'}{\rho_{00}'} \\ p = \frac{p'}{E/\Delta^3}, \quad u = \frac{u'}{(E/\rho_{00}'\Delta^3)^{1/2}}, \quad E = \frac{E_0}{\alpha_0} \end{gathered} \tag{1.2}$$

where the dimensionless factor α_0 depends [10, 11] on the coefficient of the adiabatic curve γ, assumed constant.

The equations describing the motion assume the form /26

$$\begin{gathered} \frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + v\frac{\partial\rho}{\partial z} + \rho\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(ru)\right) = 0 \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \qquad \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + A_g = 0 \\ \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + v\frac{\partial p}{\partial z} + \gamma p\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(ru)\right) = 0, \qquad A_g = \frac{g\rho_{00}'\Delta^4}{E} \end{gathered} \tag{1.3}$$

The boundary conditions for the shock wave will be those of Rankine and Gugonio:

$$\begin{gathered} \rho = e^{-z}\frac{\gamma+1}{\gamma-1}\left[1 + \frac{2\gamma}{\gamma-1}\frac{A_p}{N^2}\right]^{-1}, \qquad p = e^{-z}\frac{2}{\gamma+1}N^2\left[1 - \frac{\gamma-1}{2}\frac{A_p}{N^2}\right] \\ u = \frac{2}{\gamma+1}N\cos\sigma\left[1 - \gamma\frac{A_p}{N^2}\right], \quad v = \frac{2}{\gamma+1}N\sin\sigma\left[1 - \gamma\frac{A_p}{N^2}\right], \quad A_p = \frac{p_{00}'\Delta^3}{E} \end{gathered} \tag{1.4}$$

where N is the rate of propagation of the shock wave, and σ is the angle of the normal to the front with axis r. The following conditions of symmetry are imposed on the axis $r = 0$:

$$u = 0, \qquad \frac{\partial p}{\partial r} = \frac{\partial \rho}{\partial r} = \frac{\partial v}{\partial r} = 0 \tag{1.5}$$

The explosion is assumed to be strong, and the parameters A_g and A_p in (1.3) and (1.4), representing the influence of the force of gravity and counterpressure, are assumed to be equal to zero. The solution of the problem obtained in this manner depends on a single dimensionless parameter, the index of the adiabatic curve γ.

2. Method of Solution. In the half plane Π a central area G_0 is marked off (referred to in future as c.a.), with a boundary $G_0(t)$ containing the point of the explosion P_0. The boundary $G_0(t)$ is selected in the course of solving the problem in such a way that the pressure over the entire c.a. may be considered constant. The physical basis for this assumption is the high speed of propagation produced in the vicinity of the point P_0.

For each moment, the pressure in the c.a. is determined with the aid of the energy balance; the density and velocity that are required for calculating the kinetic energy are extrapolated in the c.a. from the region of differential calculation G_1, bounded by the curve $G_0(t)$, the front of the shock wave $G_1(t)$ and two segments of the axis of symmetry (Figure 1).

With the aid of a special system of coordinates represented schematically in Figure 1, area G_1 is transformed into a fixed rectangle in the plane of the calculated variables (ξ, ϑ). The motion equation (1.3), transformed to the variables (τ, ξ, ϑ), is approximated with the aid of a clear two-level system first used by G. S. Roslyakov and L. A. Chudov in 1962 to solve the problem of supersonic flow around a blunt body [12].

To calculate the shock wave front under the conditions of Rankine and Gugonio, an equation was formulated for the total speed of the particles at the front. Different approximations of the conditions of symmetry (1.5) were used in the areas of the boundary corresponding to segments of the axis of symmetry.

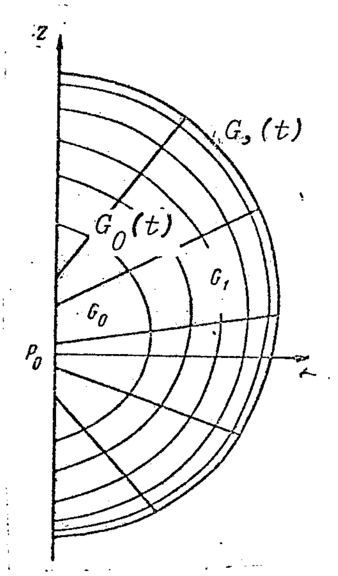

Figure 1.

Smoothing was used to damp the oscillations arising in the presence of large gradients. /27

The energy balance was used as a control.

3. Results of the Calculations. The method described was tested on various one-dimensional problems, particularly the solution of the problem of a point explosion in a homogeneous atmosphere with consideration of counterpressure. Good agreement with the data in [13] was obtained.

The solution of the problem described above was obtained on a grid consisting of 320 node points (16 rays and 20 nodes on each ray). The calculation was made at $\gamma = 1.2$. For the initial conditions in this calculation we used the self-modeling solution of L. I. Sedov [10, 11] (the initial radius of the front was 0.054). Calculations were continued until $\tau = 13.4$. The actual non-homogeneity of the problem was clearly apparent; some values varied over enormous ranges. For example, the pressure in the upper part of the front decreased in comparison to the initial value by six or seven orders of magnitude. The spatial non-homogeneity was also great: at $\tau = 13.4$ the pressure in the lower part of the front was greater than the pressure in the upper part of the front by a factor of 50. The development of non-homogeneity was accompanied by a noticeable contraction of the c.a. in the relative coordinate; the influence of non-homogeneity appears to penetrate the central zone. The pressure in the c.a. up to $\tau = 0.2$ coincides with the corresponding pressure in the self-modeling solution, then becomes smaller: at $\tau = 1.39$, the difference δp_0 is 9%, at $\tau = 4.4$, $\delta p_0 = 26\%$, etc. Evidently, in the strong stage there is a phenomenon of "suction" of particles out of the central area. A sharper drop of pressure in the c.a. in comparison with

[10, 11] evidently makes the c.a. float like an area of constant pressure observed with an increase of non-homogeneity. The shock wave, moving upward at $\tau \sim 6.2$ reaches the minimal value of its speed; dispersal of the upper part of the front then begins.

As the spatial non-homogeneity increases, so do the errors in approximation caused by a rather coarse grid, which show up as the growth of the relative imbalance of energy δE; thus, δE = 7, 20, 30% for τ = 1.39, 4.4, and 6.1, respectively.

The results related to large values of τ have only a qualitative value. /28 Figure 2 shows for τ = 6.1 the distributions of the functions p and ρ on coordinate z along the lower and upper rays of the grid. At this moment the shock wave front travels upward a distance which is more than two factors greater than the corresponding distance downward. Large gradients of the solution in the lower part and a complete transformation of the density profile as well as the main pressure in the upper part of the excited area are noted. In addition, there is a qualitative correspondence of these profiles to the self-modeling solution [5].

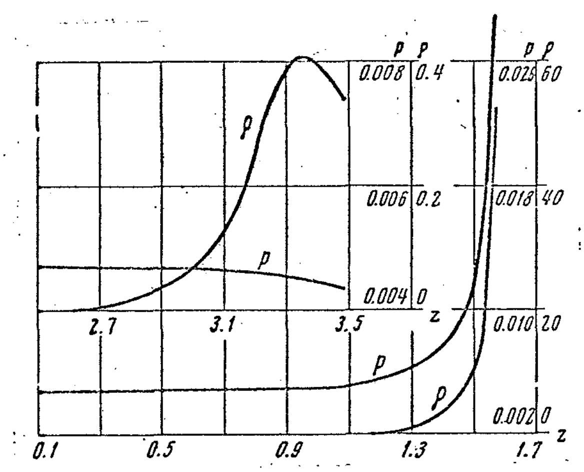

Figure 2.

4. Comparison with Data in [2]. The positions of the shock wave front on the basis of the results of this work are shown in Figure 3 for τ = 3.05 and 6.1 (solid curves); the data from [2] are represented by dots. The maximum relative deviation (in the radial direction) occurs at the lower point of the axis of symmetry and does not exceed 7%.

As expected, the gas dynamic parameters in the front, which have a certain degree of sense in the conditional (integral) sense in [2], have much worse agreement with the results of our calculations. Figure 4 shows the pressure distribution in the front as a function of the angular coordinate ϑ at $\tau = 1.39$ and $\tau = 4.4$. The solid curves are plotted on the basis of our results, and points represent the results from [2].

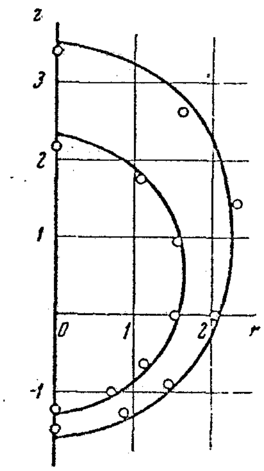

Figure 3.

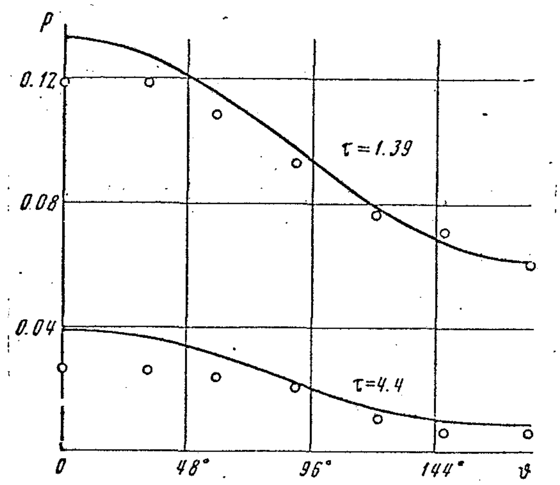

Figure 4.

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